algebra

fractions

add/sub: find a common denominator
multiply: straight across
divide: reciprocate bottom, then multiply

exponents

laws of exponents

- $a^{-n} = \frac{1}{a^n}$
- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^x b^x = (ab)^x$
- $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

radicals

- $\sqrt[n]{x^m} = (\sqrt[n]{x})^m = x^{m/n}$
- EVEN ROOTS
  - $\sqrt[n]{x^n} = |x|$
  - don't need abs values if power outside root is even
- ODD ROOTS
  - $\sqrt[n]{x^n} = x$

intervals & sets

interval	number line	inequality
(or)	open circle	$<$ or $>$
[or]	closed circle	$\leq$ or $\geq$

- union (∪): combination of both sets
- intersection (∩): overlap between sets

functions

a function passes the **vertical line test**

lines

- slope: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
- ⊥ slope: $m_\perp = -\frac{1}{m}$
- point-slope form: $y - y_1 = m(x - x_1)$

piecewise functions

$$f(x) = \begin{cases} function\ 1, & domain\ 1 \\ function\ 2, & domain\ 2 \end{cases}$$

- graph each function, but erase anything that isn't included in its associated domain

function symmetry

EVEN	ODD
symmetric about origin	symmetric about y-axis
$f(-x) = f(x)$	$f(-x) = -f(x)$

function translation

Vertical Shift
- UP: add a number outside function
- DOWN: subtract a number outside function

Horizontal Shift
- RIGHT: subtract a positive number inside function (see a negative)
- LEFT: subtract a negative number inside function (see a positive)

Reflections
- x-axis: $-f(x)$
- y-axis: $f(-x)$

changing form of a function

factoring

- common factor: something all terms share
- special formulas
  - $x^2 - y^2 = (x+y)(x-y)$
  - $(x+y)^2 = x^2 + 2xy + y^2$
  - $(x-y)^2 = x^2 - 2xy + y^2$
  - $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
  - $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
- standard factoring
- factoring by grouping
  - group terms with common factors
  - remove greatest common factor from each group
  - end up with 2 factors
- long polynomial division
  - r is a root $\Leftrightarrow$ $(x - r)$ is a factor
  - divide polynomial by known factor

completing the square

changes $ax^2 + bx + c$ to $a(x-h)^2 + k$

1. factor out a from all terms
2. look at linear coefficient, divide by 2, square it
3. add and subtract this number
4. identify the perfect square and combine remaining numbers
5. factor a back in

conjugates

- the conjugate of $\sqrt{a} \pm b$ is $\sqrt{a} \mp b$
- multiply the top ***and*** bottom of a fraction by its conjugate to simplify

solving equations

linear equations

- move all terms with variable to one side
- factor variable if needed
- isolate variable

quadratic equations

- make sure quadratic is set equal to 0 first!
- factor, use quadratic formula, or complete the square

other types of equations

- zero factor property
- isolate variable
- cross multiply
- find common denominator

system of equations

graphically: where equations intersect
algebraically: substitution

1. solve one equation for one variable
2. sub equation 1 into equation 2 to solve for one variable
3. solve for the remaining variable

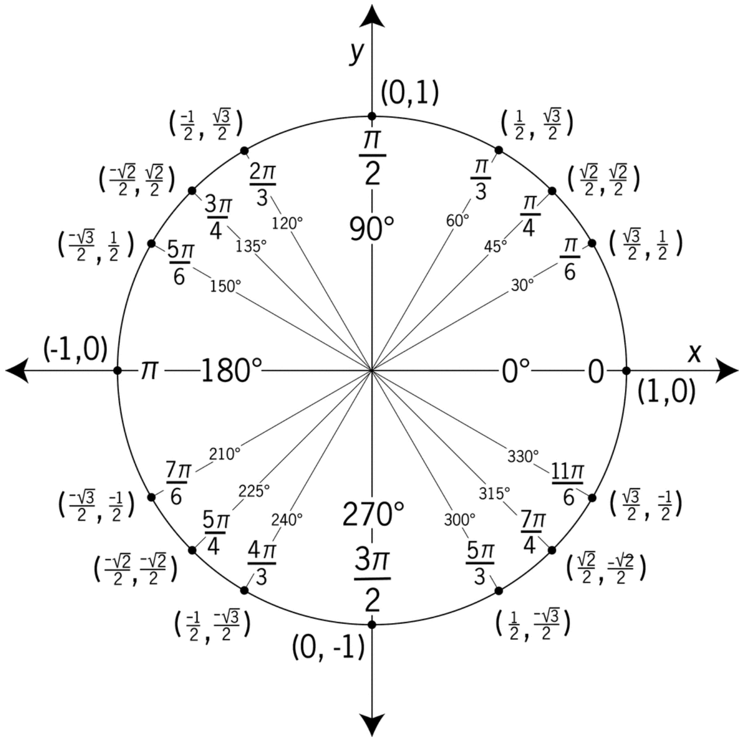
pre-calc

angles

- $^1/_2$ circle $= 180° = \pi$ radians
- **radians → degrees**: multiply by $\frac{180}{\pi}$
- **degrees → radians**: multiply by $\frac{\pi}{180}$

unit circle

$(\cos\theta, \sin\theta)$

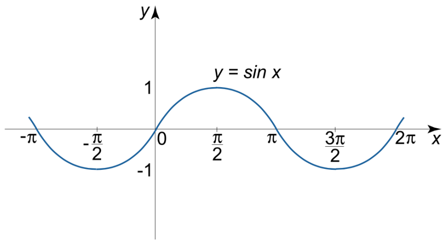


trig identities

$\cos^2\theta + \sin^2\theta = 1$

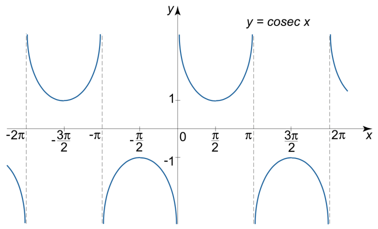
$\sin\theta$

period = 2π



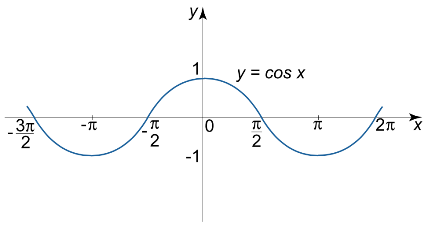
$\csc\theta$

period = 2π



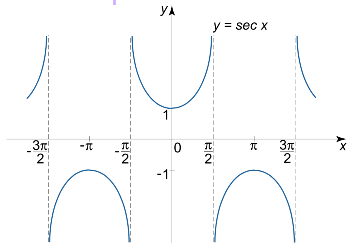
$\cos\theta$

period = 2π



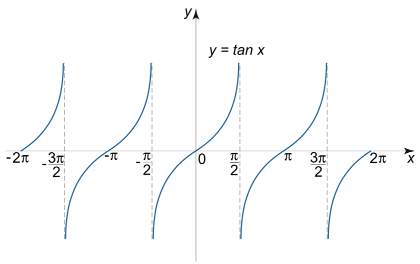
$\sec\theta$

period = 2π



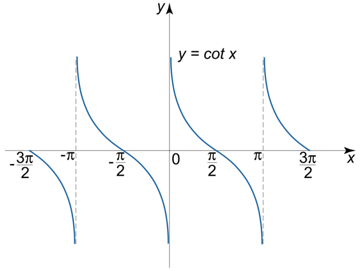
$\tan\theta$

period = π



$\cot\theta$

period = π



trig functions

input = angle
output = number

$\sin(\theta) = \#$

$\sin\theta$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta$	$\cot\theta = \frac{1}{\tan\theta}$

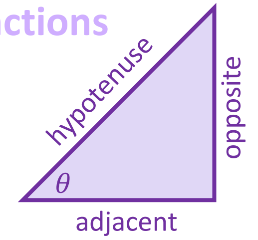
trig function symmetry

EVEN functions
- $\cos\theta$
- $\sec\theta$

ODD functions
- $\sin\theta$
- $\csc\theta$
- $\tan\theta$
- $\cot\theta$

evaluating trig functions

SOHCAHTOA



translating trig functions

Vertical Shift
- UP: add a number outside function
- DOWN: subtract a number outside function

Horizontal Shift *(phase shift)*
- RIGHT: subtract a positive number inside function (see -)
- LEFT: subtract a negative number inside function (see +)

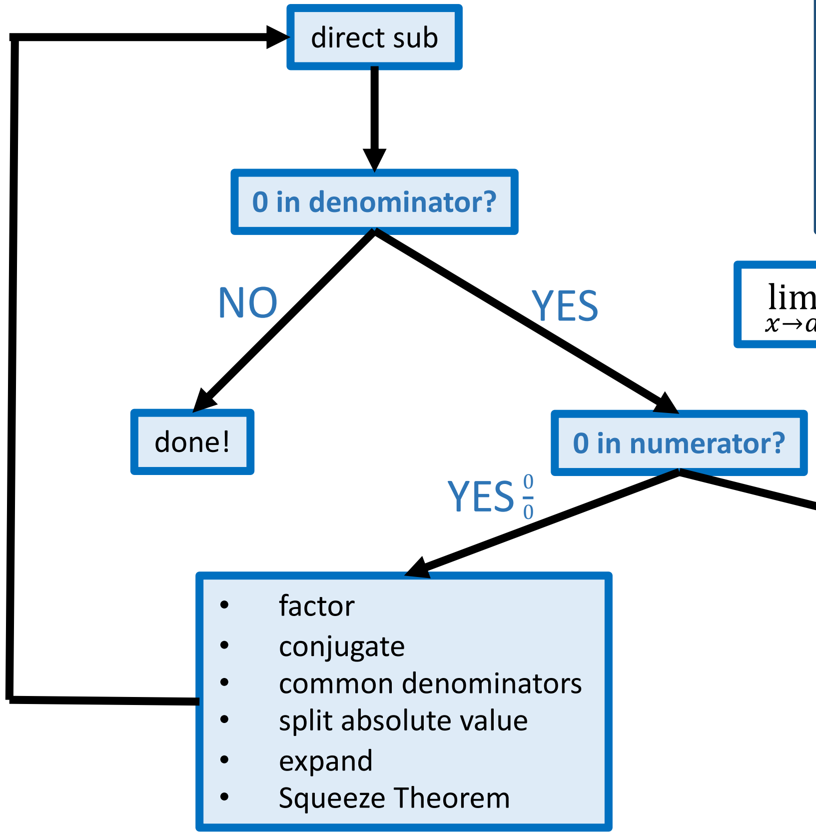
Amplitude *(how tall wave is)*
- multiplier in front of trig function; $A\sin(\theta)$

Period *(how often wave repeats)*
- multiplier inside trig functions; $\sin(B\theta)$
- new period: $\frac{original\ period}{B}$

Limits

$$\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$$

computing limits



Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

1. Begin with an inequality, $f(x) \le g(x) \le h(x)$
   ($g(x)$ is the function you want to figure out the limit of)
2. Take the limit of the outer two functions.
3. If the limits of the outer two functions evaluate to the same number, the limit of the middle function also evaluates to the same number.

infinite limits

$$\lim_{x\to a} f(x) = \infty$$

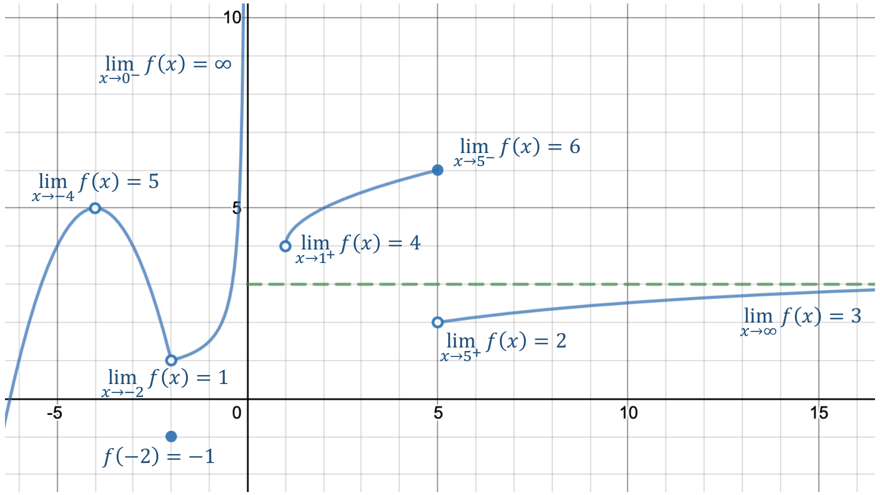
computing infinite limits

- look at left and right sided limits
- find what is causing 0 in denominator
- replace with **small +** or **small –**
- $\frac{+}{+} = \infty$, $\frac{-}{-} = \infty$, $\frac{+}{-} = -\infty$, $\frac{-}{+} = -\infty$

vertical asymptotes

1. Find the possible VA by solving denominator = 0
2. Verify $x = a$ is a VA with at least one infinite limit

limits graphically



limits at infinity

$$\lim_{x\to \pm\infty} f(x) = L$$

- $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$
- n is even, $\lim_{x\to\pm\infty} x^n = \infty$
- n is odd, $\lim_{x\to\infty} x^n = \infty$, $\lim_{x\to -\infty} x^n = -\infty$

computing limits at infinity

1. choose highest power in denominator
2. divide each term by highest power
3. simplify each term
4. evaluate each term

caution!

$x \to \infty$: $\sqrt{x^2} = |x| = x$

$x \to -\infty$: $\sqrt{x^2} = |x| = -x$

need negative sign when $x \to -\infty$ and you have an odd exponent outside radical!

horizontal asymptotes

$y = L$ is a HA if $\lim_{x\to\infty} f(x) = L$ and/or $\lim_{x\to -\infty} f(x) = L$

slant asymptotes

degree of numerator = degree of denominator + 1
long polynomial division

inverses

function composition

$f(g(x))$: $g(x)$ is the inner function and $f(x)$ is the outer function

exponential functions

$f(x) = b^x$

$b > 1$ and $b \neq 1$

domain: $(-\infty, \infty)$ **range:** $(0, \infty)$

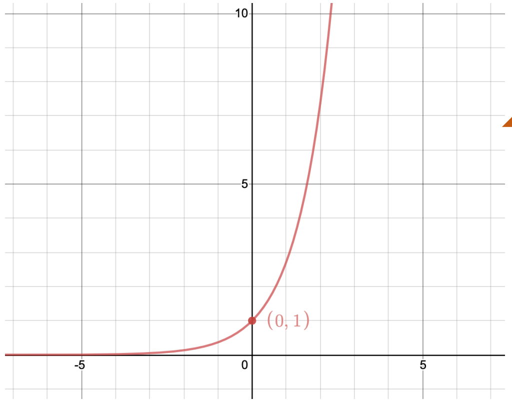
laws of exponents

- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
- $a^x = a^y \Leftrightarrow x = y$

natural exponential

$f(x) = e^x$

$e \approx 2.7183$



$b^{\log_b(x)} = x$

inverse functions

properties of inverses

- a function has an inverse if it is **one-to-one** *(passes horizontal line test)*
- $f(x)$ and $f^{-1}(x)$ reflect over the line $y = x$
- $f(x)$ and $f^{-1}(x)$ swap domains and ranges
- $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

finding an inverse

1. solve for x
2. interchange x and y
3. replace y with inverse notation, $f^{-1}(x)$

$\log_b(b^x) = x$

logarithmic functions

$f(x) = \log_b x$

domain: $(0, \infty)$ **range:** $(-\infty, \infty)$

laws of logs

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(x^r) = r\log_b(x)$
- $\log_b(b) = 1$

compute logs

$\log_b x = y \Leftrightarrow b^y = x$

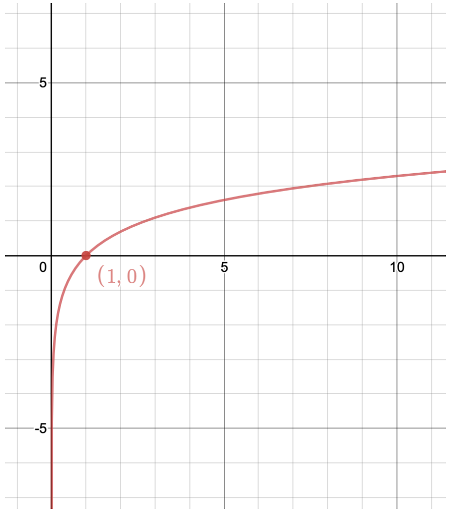
"What power do I need to raise my base to in order to get x?"

natural log

$f(x) = \ln(x)$

base e

all laws of logs also apply to $\ln(x)$!



derivatives

the derivative is the slope of the tangent line

derivative at a point

the slope of the tangent line at a single point, $x = a$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

continuity & differentiability

continuity conditions

- $f(a)$ defined
- $\lim_{x \to a} f(x)$ exists
- $f(a) = \lim_{x \to a} f(x)$

discontinuities

- removable
- jump
- infinite
- oscillating

Intermediate Value Theorem

If f is continuous on $[a, b]$ and $f(a) < L < f(b)$,
then there is a number $c \in (a, b)$ such that $f(c) = L$

differentiability $\Rightarrow$ continuity
continuity $\not\Rightarrow$ differentiability (i.e. corner or cusp)

differentiability

differentiable = able to find a derivative

a function will **not** be differentiable at a point if there is a …

- discontinuity
- corner
- cusp
- vertical tangent

derivative graphs

$f(x)$	$f'(x)$
increasing	above x-axis
decreasing	below x-axis
smooth min/max	crosses x-axis
constant	zero (over interval)
linear	constant (slope of line)
quadratic	linear

derivative applications

physics

- **position**: $s(t)$
- **velocity:** $v(t) = s'(t)$
- **acceleration**: $a(t) = v'(t) = s''(t)$

MAX HEIGHT: when v$(t) = 0$; plug time into $s(t)$
VELOCITY ON GROUND: when $s(t) = 0$; plug time into $v(t)$
SPEED: speed = $|v(t)|$
SLOWING DOWN: $v(t)$ and $a(t)$ opposite signs
SPPEDING UP: $v(t)$ and $a(t)$ same signs

derivative as a function

the slope of the tangent line *anywhere* on the original function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

given $f'(x)$, you can find the slope of the tangent line *anywhere* you'd like by plugging in a value for x

derivative rules

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
- $\frac{d}{dx}(e^x) = e^x$

- **product rule**: $\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$
- **quotient rule**: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$

- **chain rule**: $\frac{d}{dx}\big(f(g(x))\big) = f'(g(x)) \cdot g'(x)$