JIT 1.1, 1.2, 1.3

- Adding and Subtracting Fractions
  - You have to get a common denominator before adding/subtracting fractions
- Multiplying Fractions
  - multiply numerators straight across
  - multiply denominators straight across
- Dividing Fractions
  - Reciprocate (flip) bottom fraction and multiply
- Parenthesis
  - when a value or a negative sign is in front of a parenthesis, that value distributes to everything inside parenthesis

JIT 1.4, 1.5, 1.8

- Rules for exponents:

$a^{-n} = {}^{1}/_{a^n};\ a \neq 0$	A negative exponent belongs on the other side of the fraction
$a^0 = 1;\ a \neq 0$	Anything raised to the 0 power is 1
$a^x \cdot a^y = a^{x+y}$	Multiplying with the same base → add exponents
${}^{a^x}/_{a^y} = a^{x-y}$	Dividing with same base → subtract exponents
$(a^x)^y = a^{xy}$	exponent raised to an exponent → multiply exponents
$a^x \cdot b^x = (ab)^x$	exponent can "distribute" when multiplying with different bases and same exponent
${}^{a^x}/_{b^x} = ({}^{a}/_{b})^x$	exponent can "distribute" when dividing with different bases and same exponent

- Roots
  - $\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m = x^{m/n}$
- Intervals, Number lines, and Inequalities

Interval	Number line	Inequality
(or)	open circle	$<$ or $>$
[or]	closed circle	$\leq$ or $\geq$

- Set Notation
  - Union ($\cup$): combination of both sets
  - Intersection ($\cap$): overlap between two sets

JIT 4.1, 4.2

- a function should pass the vertical line test

parent	domain	range
$y = x$	$(-\infty, \infty)$	$(-\infty, \infty)$
$y = \|x\|$	$(-\infty, \infty)$	$[0, \infty)$
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = x^3$	$(-\infty, \infty)$	$(-\infty, \infty)$
$y = e^x$	$(-\infty, \infty)$	$(0, \infty)$
$y = \ln(x)$	$(0, \infty)$	$(-\infty, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = 1/x^2$	$(-\infty, 0) \cup (0, \infty)$	$(0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt[3]{x}$	$(-\infty, \infty)$	$(-\infty, \infty)$

- Equation of a line
  - slope: $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$
  - perpendicular slope: $m_\perp = -\frac{1}{m}$
  - point-slope formula: $y - y_1 = m(x - x_1)$
- Piecewise Functions
  - $f(x) = \begin{cases} \text{function 1} & \text{how much you show} \\ \text{function 2} & \text{how much you show} \end{cases}$

JIT 4.3, 4.4, 4.5, 4.6

- ODD Functions:
  - symmetric about origin
  - $f(-x) = -f(x)$
- EVEN Functions:
  - symmetric over y-axis
  - $f(-x) = f(x)$
- Vertical Shifts
  - UP: add number to outside of function; i.e. $y = x + 2$
  - DOWN: subtract number to outside of function; i.e. $y = x - 2$
- Horizontal Shifts
  - LEFT: subtract a negative number (see positive) inside function; i.e. $y = (x + 2)$
  - RIGHT: subtract a positive number (see negative) inside function; i.e. $y = (x - 2)$
- Reflections
  - reflect over x-axis: $-f(x)$
  - reflect over y-axis: $f(-x)$

JIT 4.7

- Solving a system of equations by GRAPHING:
  - intersection is solution
  - if lines are parallel → no solution
  - if lines are the same → infinite solutions
- Solving a system of equations by SUBSTITUTION:
  1. Choose one equation and solve it for one variable
  2. Insert what you found from Step 1 into the other equation and solve for the single variable
  3. Make sure you have solved for both variables

JIT 5.1, 5.2, 5.3, 5.5

- Radians → Degrees: multiply by $\frac{180}{\pi}$
- Degrees → Radians: multiply by $\frac{\pi}{180}$
- know unit circle
- know graphs for trig functions

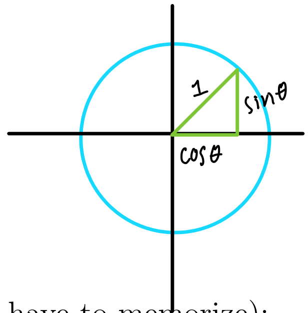
$\sin(x)$	$\csc(x) = \frac{1}{\sin(x)}$
$\cos(x)$	$\sec(x) = \frac{1}{\cos(x)}$
$\tan(x)$	$\cot(x) = \frac{1}{\tan(x)}$

- 2π period: sin, cos, csc, sec
- π period: tan, cot
- Given the value of one trig function, you can find the others using:
  - SOHCAHTOA
  - which quadrant the triangle lies in
  - Pythagorean Theorem

JIT 5.4

- $y = A\sin(B(\theta - C)) + D$
  - A: amplitude
  - B: period; divide normal period by B
  - C: horizontal shift
  - D: vertical shift

JIT 15.1



- Pythagorean Identities (memorize):
  - $\cos^2\theta + \sin^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $\cot^2\theta + 1 = \csc^2\theta$
- Addition/Subtraction Formulas (don't have to memorize):
  - $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
  - $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
  - $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
  - $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
- Double-Angle Formulas (memorize, but we don't use a lot):
  - $\sin(2x) = 2\sin(x)\cos(x)$
  - $\cos(2x) = \cos^2(x) - \sin^2(x)$ or $= 2\cos^2(x) - 1$ or $= 1 - 2\sin^2(x)$
- Half-Angle Formulas (memorize, but we don't use a lot):
  - $\cos^2(x) = \frac{1+\cos(2x)}{2}$
  - $\sin^2(x) = \frac{1-\cos(2x)}{2}$
- Solving Equation with Trig
  - get as close to angle as you can, then ask yourself, "Where does this happen on the unit circle?"

JIT 10.1, 10.2, 10.3, 10.4

- Find a common factor within all terms and take it out of each term
- Special Formulas for Factoring Quickly
  - $x^2 - y^2 = (x+y)(x-y)$
  - $(x+y)^2 = x^2 + 2xy + y^2$
  - $(x-y)^2 = x^2 - 2xy + y^2$
  - $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
  - $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
- Standard Factoring
  - ask, "What multiplies to get my last number and adds to get my middle number?"
- Factoring by Grouping
  - group terms with common factors
  - remove the greatest common factor from each group
  - you should have the same factor in each group and then what is leftover from each group is is your other factor
- Roots and Factors
  - r is a root when $f(r) = 0$
  - r is a root $\iff$ $x - r$ is a factor
  - if you already know one factor, you can use long polynomial division to find your other factors
- Completing the Square
  - factor out the number in front of x^2 (a)
  - take the coefficient on x, divide it by 2, and square it
  - this will be what you need to add and subtract
  - complete the square
  - factor a back in

JIT 10.5, 10.6

- Rationalizing with Conjugates
  - CONJUGATE: change the sign in the middle of two terms where one involves a square root
  - multiply the top and bottom of rational function by the conjugate
- Pulling things out from Radicals
  - split the stuff under the radical into factors you can pull out from under the root
  - when you have an EVEN root ($\sqrt{}$, $\sqrt[4]{}$, $\sqrt[6]{}$, ...), if you pull out something with an odd exponent it needs absolute values around it
  - when you have an ODD root ($\sqrt[3]{}$, $\sqrt[5]{}$, $\sqrt[7]{}$, ...), you don't have to worry about absolute values

JIT 3.1, 3.2, 3.3

- Techniques for Solving Equations
  - **isolate your variable** by moving all terms with your variable to one side and terms without to the other side
  - You can solve a quadratic equation by **factoring**, **quadratic formula**, or **completing the square**
    - quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
  - **Cross multiplication** can get rid of fractions on both sides of your equal sign
    - be sure to check your answer back in the original equation for 0s in the denominator
  - **common denominators** can help you get rid of fractions and allow you to just solve with the numerators
    - be sure to check your answer back in the original equation for 0s in the denominator
- Rational Functions: $y = \frac{p(x)}{q(x)}$
  - Find domain: anything that doesn't belong in domain will be when $q(x) = 0$
  - Find roots: we will cross the x-axis when $p(x) = 0$
  - Find y-intercepts: plug in 0 into entire function; $\frac{p(0)}{q(0)}$

Ch 2.1: The Idea of Limits

- **secant lines:** connect two points on a curve
  - slope: $m_{\text{sec}} = \frac{s(b)-s(a)}{b-a}$
  - average velocity
- **tangent lines:** touch the curve at a single point
  - slope: we can conjecture the slope of a tangent line by bringing the points of a secant line closer and closer together
  - instantaneous velocity

Ch 2.2: Finding Limits Graphically

- **NOTATION:** $\lim\limits_{x\to a} f(x) = L$
- Limits tell us what our functions appears to be *approaching* as x closes in around a. (aka: limits don't care about holes!)
- one-sided limits:
  - right-sided limits approach a from the right: $\lim\limits_{x\to a^+} f(x) = L$
  - left-sided limits approach a from the left: $\lim\limits_{x\to a^-} f(x) = L$
  - If $\lim\limits_{x\to a^+} f(x) = L$ and $\lim\limits_{x\to a^-} f(x) = L$, then $\lim\limits_{x\to a} f(x) = L$
  - If $\lim\limits_{x\to a^+} f(x) = L$ and $\lim\limits_{x\to a^-} f(x) = M$, then $\lim\limits_{x\to a} f(x)$ DNE

Ch 2.3: Finding Limits Algebraically

- **Limit Laws:**
  - $\lim\limits_{x\to a}(f(x) \pm g(x)) = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$
  - $\lim\limits_{x\to a}(cf(x)) = c\lim\limits_{x\to a} f(x)$
  - $\lim\limits_{x\to a}(f(x)g(x)) = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$
  - $\lim\limits_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (if $\lim\limits_{x\to a} g(x) \neq 0$)
  - $\lim\limits_{x\to a}(f(x))^n = \left(\lim\limits_{x\to a} f(x)\right)^n$
  - $\lim\limits_{x\to a}(f(x))^{1/n} = \left(\lim\limits_{x\to a} f(x)\right)^{1/n}$
  - $\lim\limits_{x\to a} c = c$

Ch 2.3: Finding Limit Algebraically Continued

- **Direct Substitution:**
  - *always* try direct substitution first when calculating a limit!
  - plug in a into the function. If the result is a number, this is the answer to your limit.
  - $\lim_{x\to 2}(x^2-1) = 2^2 - 1 = 4 - 1 = 3$
- **Direct Sub Yields** $\frac{0}{0}$
  - factor
  - multiply by the conjugate
  - find a common denominator
  - expand any powers
  - replace an absolute value with the appropriate part of the piecewise function
- **Squeeze Theorem**
  - If $f(x) \le g(x) \le h(x)$ for all x near a and $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.
  - start with inequality, take limit of each function, evaluate outer limits.
  - If the outer limits match, the limit of the inner function is also the same number. If not, Squeeze Theorem is inconclusive.

Ch 2.4: Infinite Limits

- $\lim_{x\to a} f(x) = \pm\infty$
- **Direct Sub Yields** $\frac{\textbf{number}}{0}$
  - Your answer will be $-\infty$ or ∞
  - Simply (factor) the function as much as possible
  - Look at the factor in the denominator that is giving you 0
  - Determine if this factor is approaching a small $+$ or a small $-$ number as x approaches a
  - $\frac{+}{+} = +$, $\frac{-}{-} = +$, $\frac{-}{+} = -$, $\frac{+}{-} = -$
- **Vertical Asymptotes**
  - $x = a$
  - Solve your denominator equal to 0
  - verify that $x = a$ is a vertical asymptote by seeing if $\lim_{x\to a^-} f(x) = \pm\infty$ **OR** $\lim_{x\to a^+} f(x) = \pm\infty$ **OR** $\lim_{x\to a} f(x) = \pm\infty$

Ch 2.5: Limits at Infinity

- $\lim_{x\to\pm\infty} f(x) = L$
- tell us the *end behavior* of a function
- **Horizontal Asymptotes**
  - If $\lim_{x\to\pm\infty} f(x)$ is a number L, then $y = L$ is a horizontal asymptote.
- Note that we can have infinite limits at infinity: $\lim_{x\to\pm\infty} f(x) = \pm\infty$
- **Polynomials**
  - $\lim_{x\to\pm\infty} x^n = \infty$ when n is even
  - $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to-\infty} x^n = -\infty$ when n is odd
  - $\lim_{x\to\pm\infty} p(x) = \lim_{x\to\pm} a \cdot x^n = \pm\infty$ depending on the degree of x^n (even or odd) and the sign of a
  - $\lim_{x\to\pm\infty} x^{-n} = \lim_{x\to\pm\infty} \frac{1}{x^n} = 0$
- **Rational Functions where numerator and denominator are *polynomials***
  - identify the highest power of x in the denominator
  - divide *every* term in the function by that highest power from the denominator
  - simplify each term
  - take the limit of each term (recall: $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$)
  - If the degree of the top polynomial is less than the degree of the bottom polynomial, $y = 0$ is a horizontal asymptote
  - If the degrees are the same, the horizontal asymptote is the fraction of the leading coefficients
  - If the degree of the top polynomial is greater than the degree of the bottom polynomial, there are no horizontal asymptotes
  - If the degree of the top polynomial is exactly one more than the degree of the bottom polynomial, there are no horizontal asymptotes, but there is a slant asymptote
    - ☐ Find the equation of a slant asymptote by long polynomial division
- **Rational Functions where numerator and denominator may *not* be polynomials**
  - identify the highest power of x in the denominator
    - ☐ Is the power even or odd when pulled outside of the radical?
    - ☐ If even outside of the radical, that power of x will always be positive.
    - ☐ If odd out outside of the radical, that power of x depends on if $x \to -\infty$ or $x \to \infty$.
  - divide *every* term in the function by that highest power from the denominator
    - ☐ Underneath any radicals, make sure you are dividing each term by the power of x that is appropriate for within the radical as it is outside the radical
    - ☐ Outside the radical, divide by the appropriate sign and power on x
  - simplify each term
  - take the limit of each term (recall: $\lim_{x\to\pm\infty} \frac{1}{x^n} = 0$)

JIT 6.1, 6.2: Exponential Functions

- $f(x) = b^x$
- If $b > 1$, our function is growing really fast.
- If $0 < b < 1$, our function is decaying really fast
- All exponential functions (without a shift) will pass through $(0, 1)$ since any number raised to the zero power (except 0) is 1.
- **Translations and Reflections**
  Let c be a positive number...
  - Horizontal Shift: b^{x-c} right shift; b^{x+c} left shift
  - Vertical Shift: $b^x + c$ upward shift; $b^x - c$ downward shift
  - Reflect over x-axis: $-b^x$
  - Reflect over y-axis: b^{-x}
- **Solving Equation with Exponential Functions**
  - If the bases are the same, you can set the exponents equal to each other and solve
- **The Natural Exponential Function**
  - $f(x) = e^x$
  - $e = 2.71828...$
  - goes through $(0, 1)$
  - slope of tangent line is 1 at $x = 0$

JIT 7.1: Composition of Functions

- Rather than a variable input, x, a function can take in another function as its input
- $(f \circ g)(x) = f(g(x))$ means to plug in $g(x)$ everywhere you see an x in your function $f(x)$.
- If $f(x) = x^2 + 2$ and $g(x) = x - 3$, $f(g(x)) = (x - 3)^2 + 2$

JIT 7.2, 7.3, 7.4: Inverse Functions

- If $f(g(x)) = x$ and $g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverses.
- We can denote the inverse of $f(x)$ as $f^{-1}(x)$
- The domain of $f(x)$ is the range of $f^{-1}(x)$
- The range of $f(x)$ is the domain of $f^{-1}(x)$
- **Graphically**
  - **Horizontal Line Test**: A function passes the HLT if you drag a horizontal line down your function and it only crosses your function at one point
  - A function is **one-to-one** if it passes the HLT
  - If a function is one-to-one, it has an inverse
  - The inverse of a function is the function reflected over the line $y = x$
  - If the point (a, b) lies on $f(x)$, the point (b, a) lies on $f^{-1}(x)$.
- **Algebraically**
  - Solve the function for x
  - Swap x and y
  - Replace y with inverse notation

JIT 8.1, 8.2, 8.3, 8.4: Logarithmic Functions

- $f(x) = \log_b(x)$
- logarithmic functions and exponential functions are inverses: $\log_b(x)$ is the inverse of b^x
  - $b^{\log_b(x)} = x$
  - $\log_b(b^x) = x$
- To evaluate what $\log_b$(number) is, ask yourself, "What power to I need to raise by base b to to get this number on the inside?"
  - ex: $\log_9(81) = 2$ because $9^2 = 81$.
- **Laws of Logs**
  - $\log_b(xy) = \log_b(x) + \log_b(y)$
  - $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
  - $\log_b(x^r) = r\log_b(x)$
- When solving an equation involving logs, make sure you check that your solution is valid!
- **The Natural Log Function**
  - $f(x) = \ln(x)$
  - log base e
  - inverse of e^x
    - ☐ $e^{\ln(x)} = x$
    - ☐ $\ln(e^x) = x$
  - $\ln(1) = 0$, passes through $(1, 0)$
- **Change of Base**
  - If you want to change from base b to base e: $b^x = e^{x\ln(b)}$ and $\log_b(x) = \frac{\ln(x)}{\ln(b)}$
  - If you want to change from base b to base c: $b^x = c^{x\log_c(b)}$ and $\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$

Ch 2.6: Continuity

- For a function to be continuous at $x = a$, we need
  - $f(a)$ defined
  - $\lim_{x\to a} f(x)$ exists
  - $f(a) = \lim_{x\to a} f(x)$
- We have 4 types of discontinuities:
  - removable
    - violates $f(a) = \lim_{x\to a} f(x)$
    - You'll have a removable discontinuity when you cancel out a factor from the numerator and denominator
    - You can fix a removable discontinuity by "filling in the hole" and making $f(a) = \lim_{x\to a} f(x)$
  - jump
    - violates $\lim_{x\to a} f(x)$ exists (b/c $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$)
  - infinite
    - violates $f(a)$ defined and sometimes $\lim_{x\to a} f(x)$ exists
  - oscillating
    - violates $f(a)$ defined and $\lim_{x\to a} f(x)$ exists

Ch 2.6: Intermediate Value Theorem

- If f is continuous on $[a, b]$ and $f(a) < L < f(b)$, then there is a number c within (a, b) such that $f(c) = L$
- "Has there ever been a time when you were *exactly* three feet?"
  - Yes because growth is **continuous** and there was a time when you were **less than** three feet ($f(a) < L$) and a time when you were **greater than** three free ($L < f(b)$). So, there must have been an instant (c) when you were exactly three feet ($f(c) = L$).
- A lot of times we use IVT to determine if something has a root/solution/crosses the x-axis.
  - check function is continuous
  - check there is a point, a, such that $f(a) < 0$
  - check there is a point, b, such that $0 < f(b)$.
  - conclude that there must be a point, c, where $f(c) = 0$

JIT 11.1: Slopes of Secant Lines

- **secant line**: goes through *two* points on a curve

$$\frac{f(x)-f(a)}{x-a} \quad \text{or} \quad \frac{f(a+h)-f(a)}{h}$$

- Techniques to simplify:
  - expansion
  - common denominator
  - conjugate
  - canceling factors

Ch 3.1: Derivatives at a Point

- **derivative at a = slope of tangent line at a = instantaneous change at a**
- If we take the limit of the slope of a secant line (push the two points together), we get the slope of a tangent line

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \qquad \text{or} \qquad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- These definitions are *interchangeable* and will give you the same numerical answer for the slope of a tangent line *at a point*.

Ch 3.2: The Limit Definition of a Derivative

- **derivative (as a function) = slope of tangent line *anywhere* on the curve**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- **Derivative Notation**: y', $f'(x)$, f', $\frac{dy}{dx}$, $\frac{d}{dx}(f(x))$
- If we graph the derivative function, we are simply graphing the *slopes of the tangent lines* to our orginal function

function graph	**derivative graph**
increasing	positive (above x-axis)
decreasing	negative (below x-axis)
smooth min/max	zero (cross x-axis)
constant	zero (over an interval)
linear	constant
quadratic	linear

- A function will not be differentiable at a point (can't find derivative) if it has the following at that point:
  - discontinuity
  - sharp point (corner or cusp)
  - vertical tangent (supa steeeeep)

- If a function is differentiable at a, then it is continuous at a.

EQUATION OF A TANGENT LINE

- We need three things to find the equation of a line:
  - slope (m)
  - x-point (x_1)
  - y-point (y_1)
- If we are finding the equation of a tangent line, the slope of the tangent line will be the derivative.
- To find the slope of the tangent line at our x_1 value, we plug in x_1 into the derivative.
  - $m_{tan}\Big|_{x_1} = f'(x_1)$
- To find the y_1 value, we plug in our x_1 value into the original function
  - $y_1 = f(x_1)$
- To find the slope of a **normal** line, first find the slope of the tangent line, then flip and negate it.
  - $m_{norm} = -\frac{1}{m_{tan}}$

Ch 3.3: Derivative Rules

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$
- $\frac{d}{dx}(e^x) = e^x$

- We can rewrite how functions look so we can use power rule:
  - $\frac{1}{x^n} = x^{-n}$
  - $\sqrt[m]{x^n} = x^{n/m}$
- a function has a *horizontal tangent line* when $f'(x) = 0$ because the slope of a horizontal tangent line is zero
- We can find higher order derivatives by taking the derivative of the previous derivative.
  - second derivative: y''
  - third derivative: y'''
  - fourth derivative: $y^{(4)}$
  - n^{th} derivative: $y^{(n)}$

Ch 3.4: Product Rule and Quotient Rule

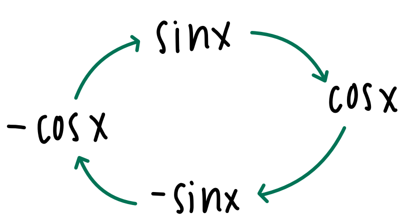
$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \qquad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

- Sometimes it's easier to simplify your function first before trying these rules.

Ch 3.5: Derivatives of Trig Functions and Special Limits

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

- Derivatives of $\sin x$ and $\cos x$ work in a cycle
  - If you want to find the n^{th} derivative of something in this cycle, divide n by 4, find your remainder, then count your remainder away from your original function



- Special limits: you need to make your coefficient and your coefficient within your angle match.

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

Ch 3.6: Applications of Derivatives

- Physics
  - position: $s(t)$ ft
  - velocity: $v(t) = s'(t)$ ft/s
  - acceleration: $a(t) = v'(t) = s''(t)$ ft/s^2
  - speed $= |v(t)|$ ft/s
  - To find an object's maximum height:
    1. First find *when* the object reaches its max height by solving where $v(t) = 0$.
    2. Then, find the max height by plugging the time you found in Step 1 into your position function, $s(t)$.
  - To find the velocity in which an object strikes the ground:
    1. First find *when* an object strikes the ground by solving where $s(t) = 0$.
    2. Then, find the velocity it strikes the ground by plugging the time you found in Step 1 into your velocity function, $v(t)$.
  - Velocity's sign signifies direction. An object possibly changes direction when $v(t) = 0$
    - ∗ If $v(t) < 0$, the object is moving down or to the left
    - ∗ If $v(t) > 0$, the object is moving up or to the right
- Economics
  - Cost Function: $C(x)$ how much to produce the first x items
  - Average Cost: $\bar{C}(x) = \frac{C(x)}{x}$ the average cost to produce x items
  - Marginal Cost: $C'(x)$ about how much it will make to produce *one more item* after making x items

JIT 12.1: Decomposition of Functions

- In a composition of functions, we have an outer and an inner function: $f(g(x))$
  - outer: $f(x)$
  - inner: $g(x)$
- We can also have a composition of three functions: $f(g(h(x)))$
  - outer: $f(x)$
  - inner: $g(x)$
  - most inner: $h(x)$

Ch 3.7: Chain Rule

- Chain Rule tells us how to take the derivative of a composition of functions
  - $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$
- Version 1
  1. First, identify your outer and inner functions.
  2. Call your inner function u and let your outer function be a function of u.
     - outer: $y = f(u)$
     - inner: $u = g(x)$
  3. Take the derivative of your outer and inner functions
     - outer derivative: $\frac{dy}{du} = f'(u)$
     - inner derivative: $\frac{du}{dx} = g'(x)$
  4. Multiply the derivatives together
     - $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u) \cdot g'(x)$
  5. Replace any us with $u = g(x)$ so that your final answer is only in terms of x.
     - $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$
- Version 2
  1. First, identify your outer and inner functions.
  2. Take the derivative of your first, leaving your inside function *unchanged*.
     - $f'(g(x))$
  3. Multiply by the derivative of the inside function
     - $f'(g(x)) \cdot g'(x)$
- You can use Chain Rule as many times as needed to ensure you take the derivative of your entire composition of functions.